Unified Torque Expressions of AC Machines

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Outline

1. **Review of torque calculation methods.**

2. **Interaction between two magnetic fields.**

3. **Unified torque expression for AC machines.**

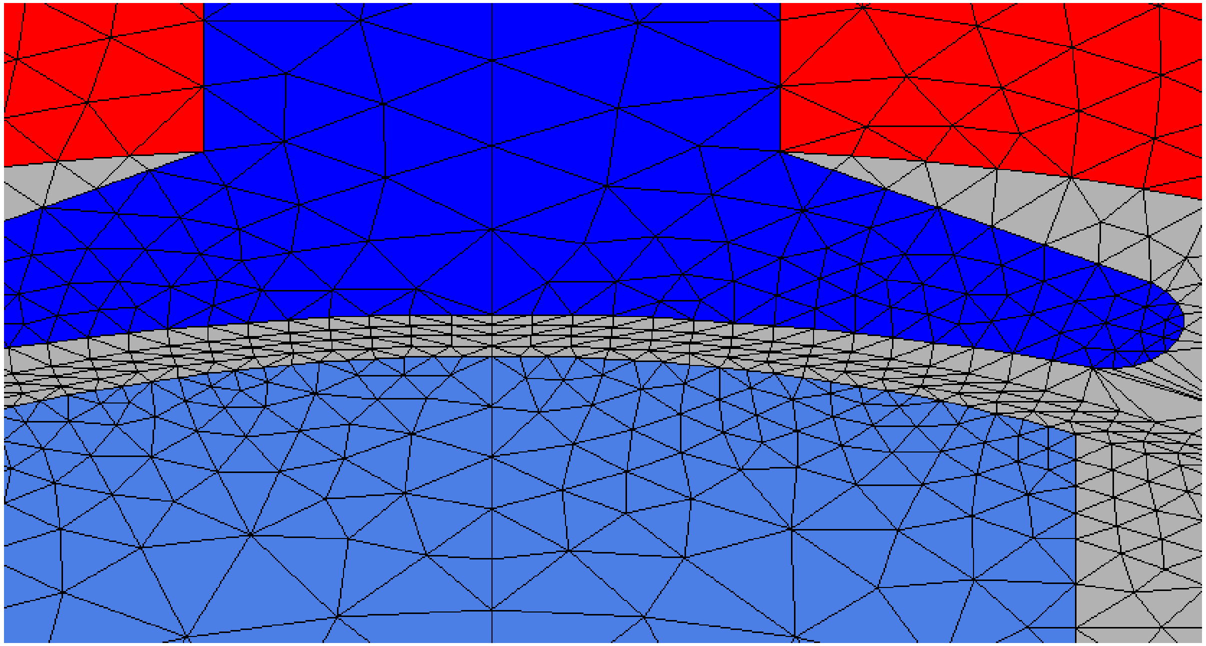
   - Permanent Magnet (PM) machine;
   - Synchronous Reluctance Machine (SynRM);
   - Induction Machine (IM);

4. **Conclusion.**

Torque Calculation Methods

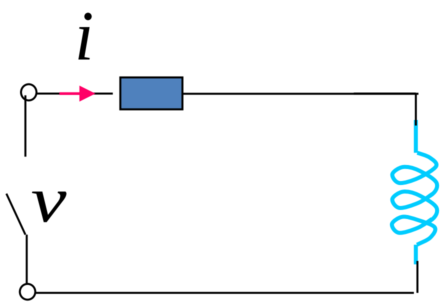
Numerical method with FEA.

Maxwell stress tensor ($\frac{B_n \cdot B_t}{2\mu_0}$) in the air gap region.



Torque Calculation Methods

Based on the energy conversion theory



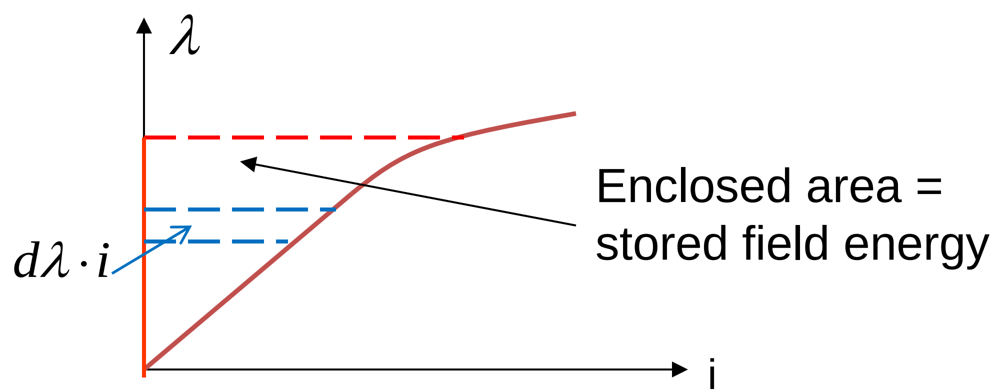
$$u = Ri + \frac{d\lambda}{dt} \qquad \lambda = Li$$

$$\Downarrow$$

$$ui = Ri^2 + \frac{d\lambda}{dt} i$$ Power balance

$$\Downarrow$$

$$uidt = Ri^2 dt + d\lambda i$$ Energy balance



Similar for an electrical machine

$$u_{qs} = R_s i_{qs} + \frac{d}{dt}\lambda_{qs} + \omega_{r,el}\lambda_{ds} \qquad u_{ds} = R_s i_{ds} + \frac{d}{dt}\lambda_{ds} - \omega_{r,el}\lambda_{qs}$$

For example, the q-axis, stator side winding analysis:

$$\boxed{P_{inq}} = i_{qs}u_{qs} = \boxed{R_s i^2{}_{qs}} + \boxed{i_{qs}\frac{d}{dt}\lambda_{qs}} + \boxed{\omega_{r,el}\lambda_{ds}i_{qs}}$$

Copper loss

Rate change of the stored field energy

Output mechanical power

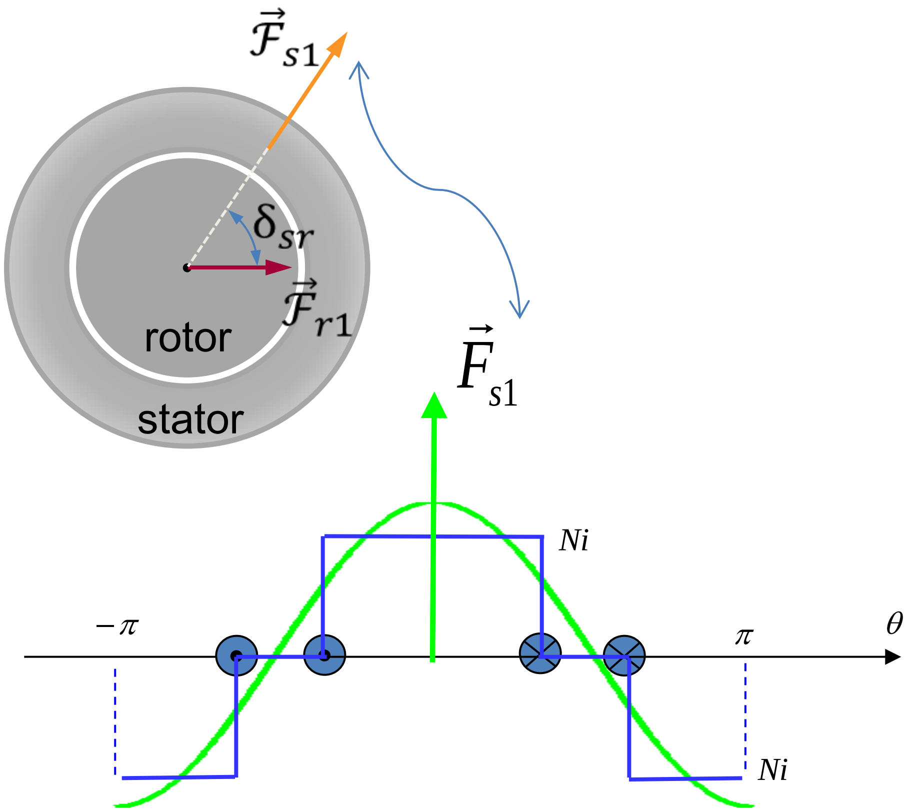
Input power of the q-axis winding

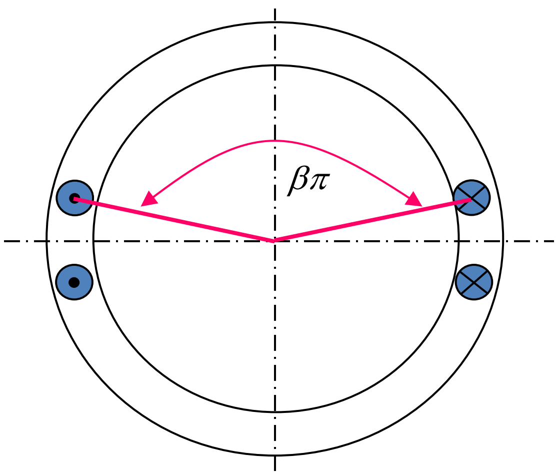
$$P_{mec,dq} = \omega_{r,el}\left(\lambda_{ds}i_{qs} - \lambda_{qs}i_{ds}\right)$$

Torque Calculation Methods

Another way to utilize the energy conversion theory

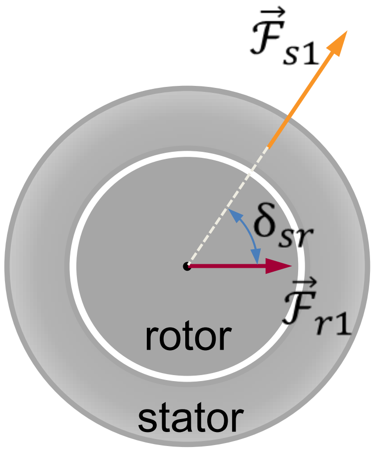
- Uniform air gap
- Sinusoidal MMF, B waveforms
- Similar MMF on the stator and rotor





Torque Calculation Methods

So finding the air gap average co-energy density



air-gap total MMF (fundamental component)

$$F_{sr1} = F_{s1}^2 + F_{r1}^2 + 2F_{s1}F_{r1}\cos\delta_{sr}$$

air-gap actual field intensity

$$H_{ag,peak} = \frac{F_{sr1}}{g}$$

Average co-energy density assuming sinusoidal air-gap field

$$= \frac{\mu_0}{2}\boxed{\frac{\left(H_{ag,peak}\right)^2}{2}} = \frac{\mu_0}{4}\left(\frac{F_{sr1}}{g}\right)^2$$

Average of sin square function gives

Torque Calculation Methods

So the torque is obtained as

By accounting the volume of the air, the total co-energy

$$W_{co,ave} = \frac{\mu_0}{4}\left(\frac{F_{sr1}}{g}\right)^2 \cdot \pi DLg$$

By differentiation the co-energy, we obtain the torque

$$T = k_T \mathcal{F}_{s1} \mathcal{F}_{r1} \sin \delta_{sr}$$

$$\boxed{k_T = \frac{\mu_0 \pi DL}{2g} p}$$

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Torque expression for comparison

1. Some observations

- Classical torque equation may involve different inductances and e.g. PM flux linkage – direct comparison is not so obvious
- We experience winding current excited magnetic field (MMF), permanent magnet field (PMSM) and salient rotor magnetic field modulation effects (sync. Reluctance motor).

2. An ideal torque expressions for AC machines.

- Applying the same principle.
- Intuitive understanding of torque production mechanism.
- Torque expressions with the same geometrical parameters and physical quantities.

Torque Calculation Methods

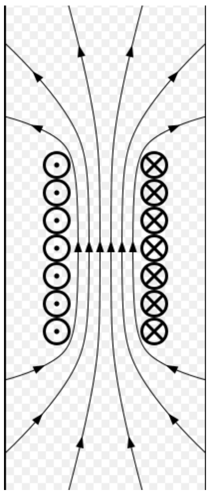
So steps to take

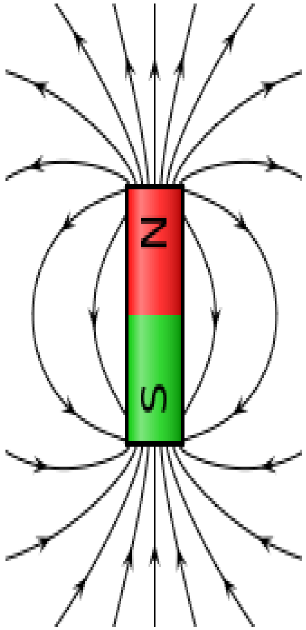
- Turn all other magnetic field into winding current excited magnetic field (MMF)

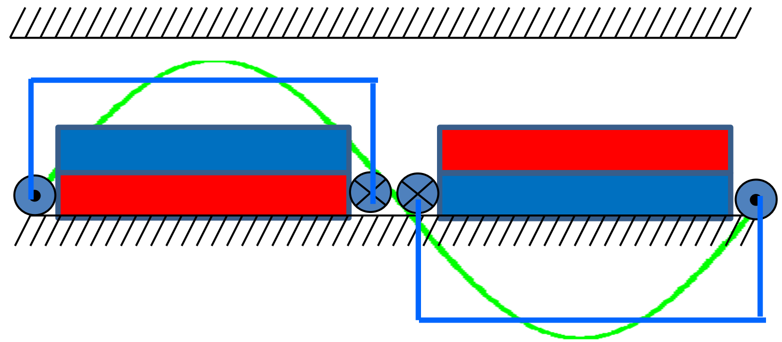
- Using a uniform airgap

Magnet equivalent magnetic field

Consider a permanent magnet





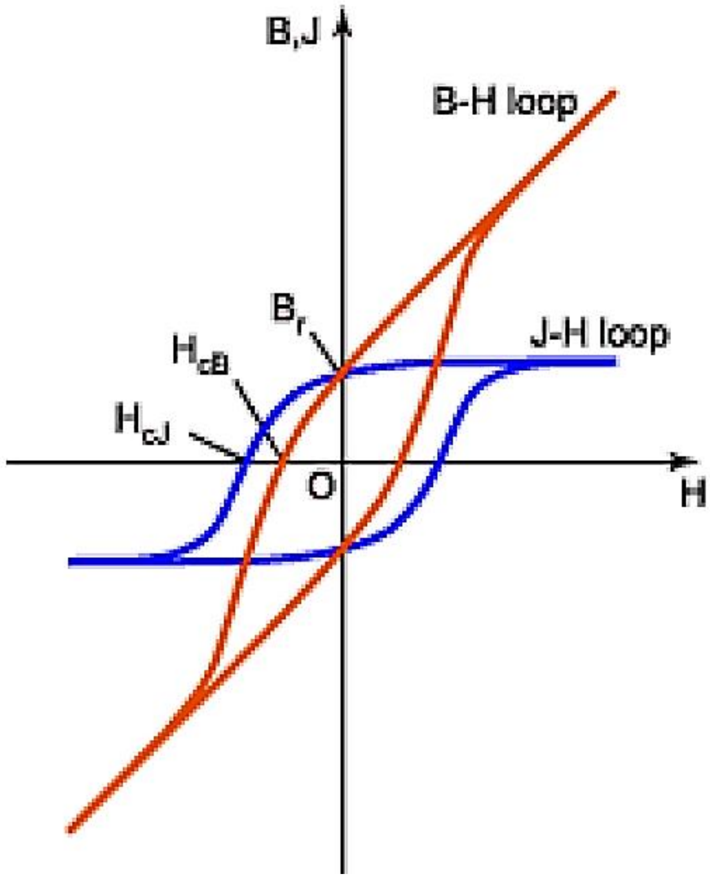


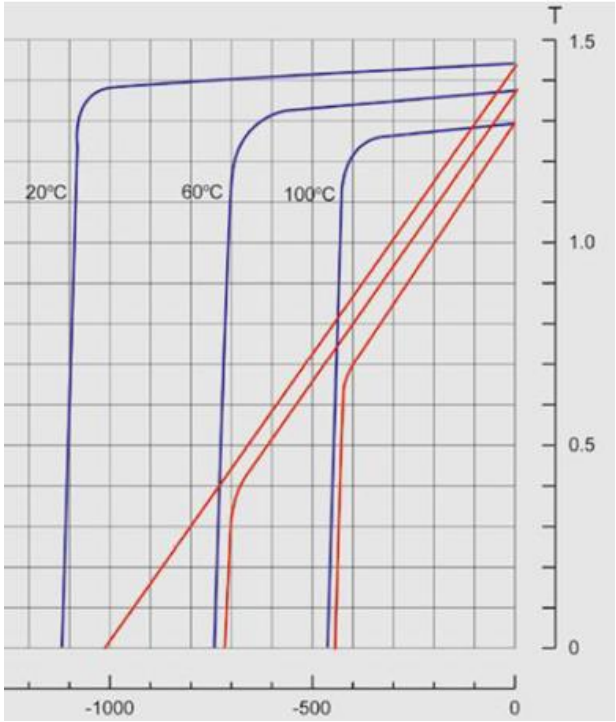
Air gap B waveform or winding MMF waveform

It is possible to replace the magnet with winding MMF for producing the same air gap B waveform

Magnet equivalent magnetic field

For a magnet





J and B have the same unit: [T]

The relationship

$$J = B - \mu_0 H$$

$$B = B_r + \mu_{pm} H$$

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Magnet equivalent magnetic field

Convenient expression

$$J = B + \mu_0 H \xrightarrow{B = B_r - \mu_{pm} H} J = B_r - \left(\mu_{pm} - \mu_0\right) H$$

$$\Big\downarrow \; J = \mu_0 M$$

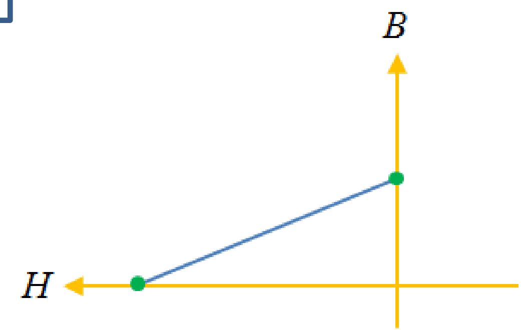
$$B = \mu_0 M - \mu_0 H$$

M: moment (not a constant as well)

$M = \dfrac{B_r}{\mu_0}$ *at H = 0*

J is not a constant; slightly decrease when H increases

(Only μ_0 *is involved)*



Positive H value definition

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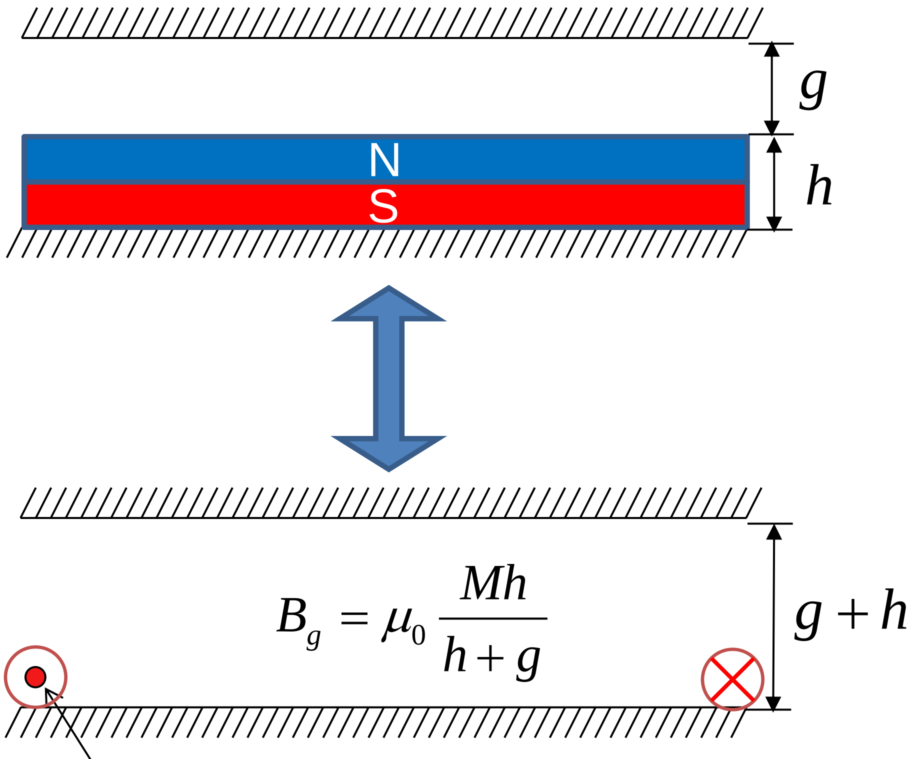
Magnet equivalent magnetic field

The PM magnetic field



$$B_g = \mu_0 \frac{Mh}{h+g}$$

$$MMF = Mh$$

For example:

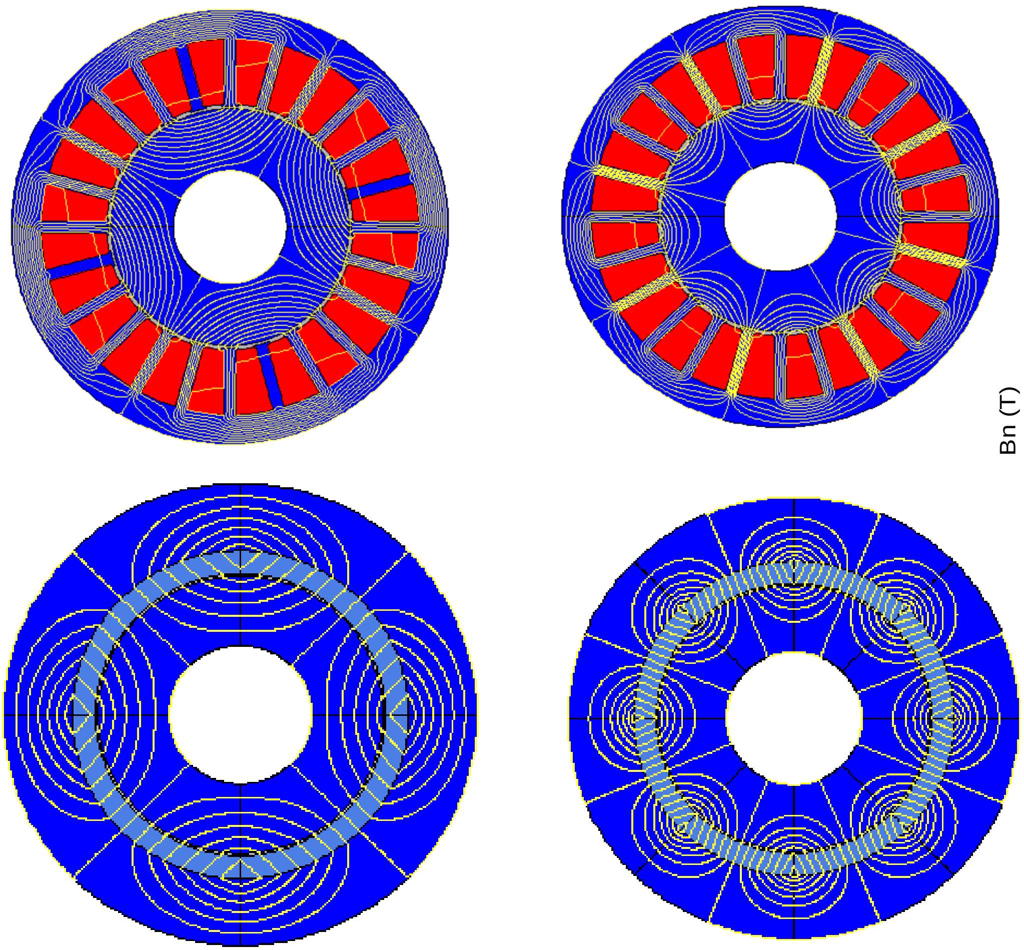
h = 1 mm
Br = 1.2 (T)
Mh = 955 (A.turns)

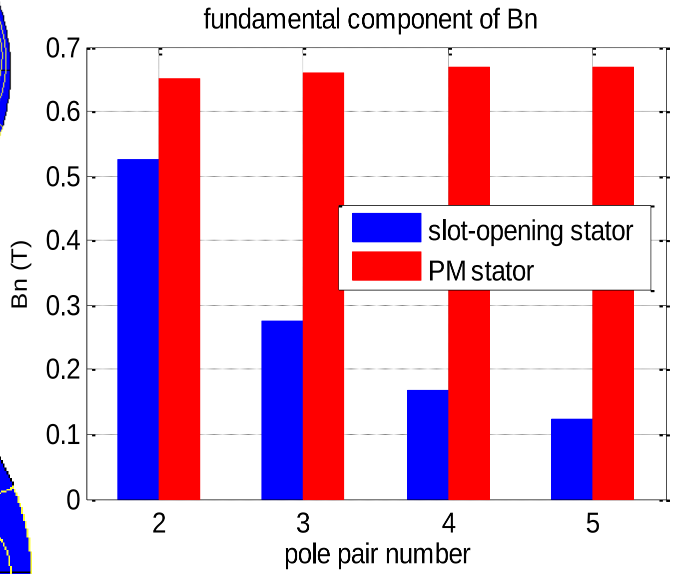
$$M = \frac{B_r}{\mu_0}$$

The MMF does not decrease when the magnet becomes narrower (neglecting the leakage flux)

Magnet equivalent magnetic field

Magnet MMF vs. winding MMF

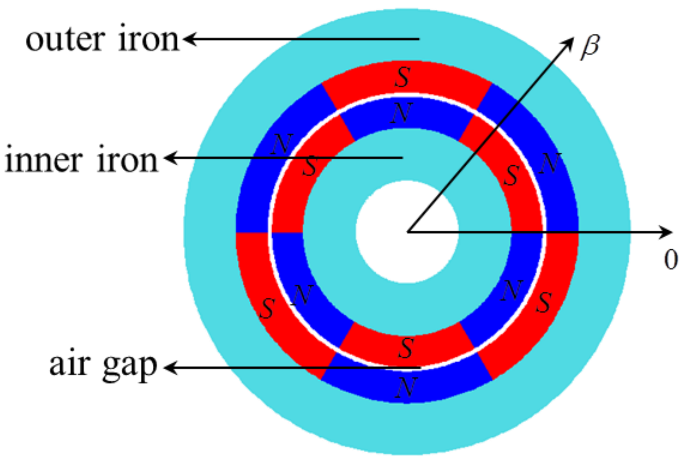


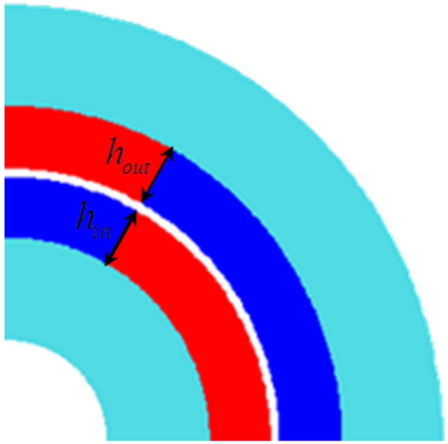


Magnetic Coupling (MC)

1. Torque production of magnetic coupling could be typically explained with the interaction between two magnetic fields.

2. Models:





4. Magnetic field from permanent magnets (fundamental component)

$$\text{Inner PMs: } B_{n1\_in} = k_B \mu_0 \frac{M h_{in}}{g + h_{in} + h_{out}} \sin(P\theta - \omega t + \beta_{in})$$

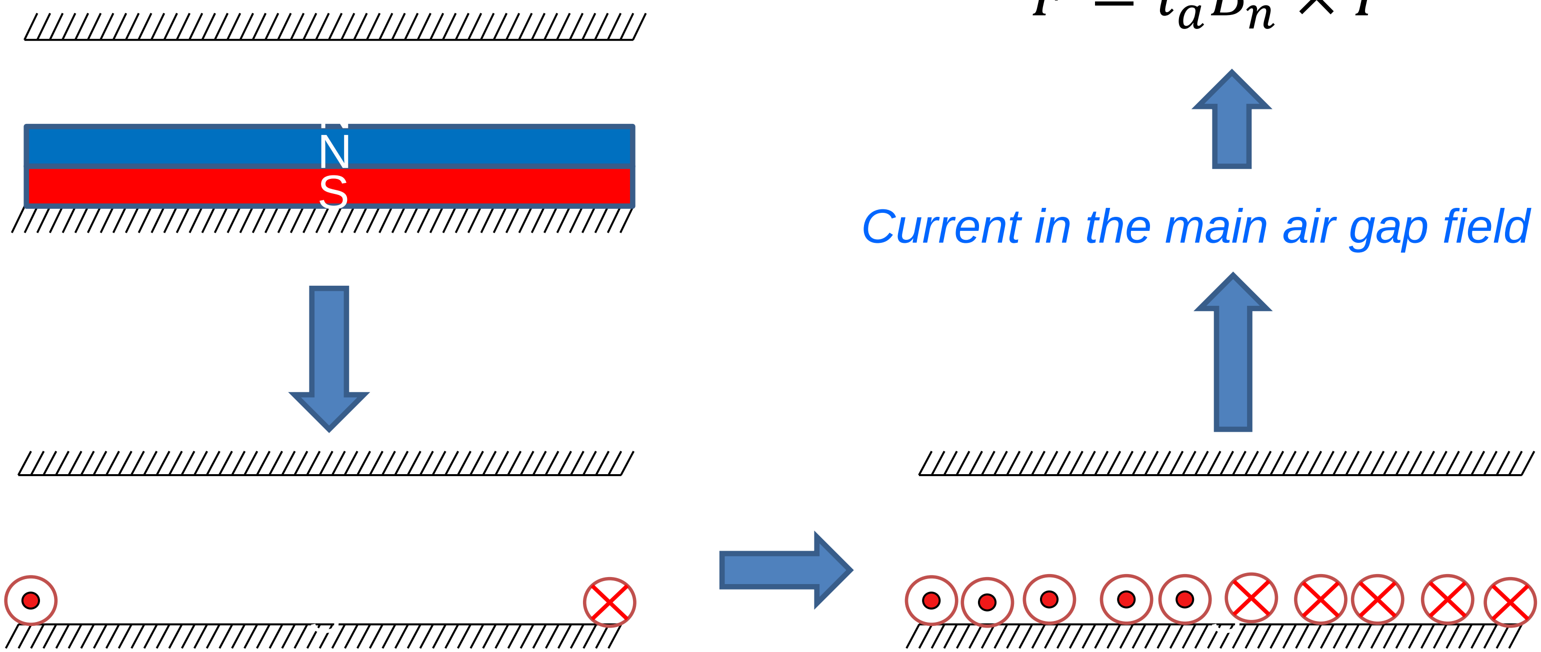
$$\text{Outer PMs: } B_{n1\_out} = k_B \mu_0 \frac{M h_{out}}{g + h_{in} + h_{out}} \sin(P\theta - \omega t + \beta_{out})$$

M: magnetization intensity; k_B: B_{n1}waveform factor considering the magnetizing direction of PMs.

Magnet equivalent magnetic field

The PM magnetic field

$$F = l_a B_n \times I$$



Distributed current density layer

Torque Evaluation of MC

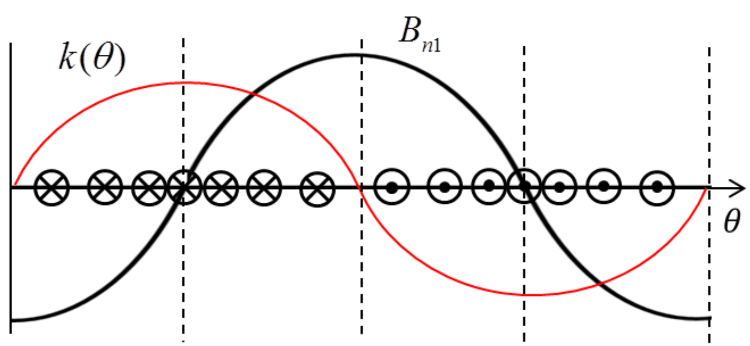
2. Equivalent current $k(\theta)$ to replace the PMs on out side of MC:

satisfying $\int k(\theta)\, d\theta = \frac{B_{n1}(\theta)}{\mu_0}(g + h_{in} + h_{out})$ (ensuring identical MMF)

yielding $k(\theta) = k_B PM h_{out} \cos(P\theta - \omega t + \beta_{out})$ (Peak value $k_m(\theta) = k_B PM h_{out}$)

Thus:

- Equivalent current is sinusoidally distributed along the inner surface of outer iron.
- Equivalent current is rotating at the same speed of PM in space.
- Peak value of equivalent current is determined by the MMF of the magnetic field source and the pole pair number.



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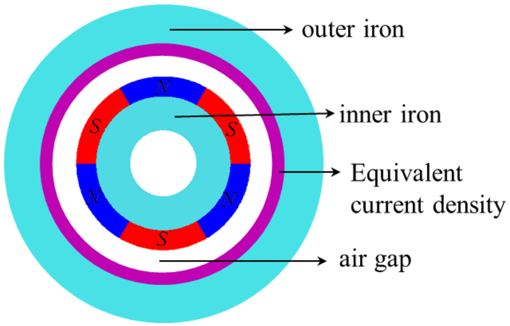
Torque Evaluation of MC

1. Equivalent model: current is located in the air gap magnetic field.



2. Electromagnetic force endured on the current at the position of θ.

$$f(\theta) = L_a B_{n1}(\theta) k(\theta)$$ (along the tangential direction)

3. Torque of MC:

$$\boldsymbol{T = \int_0^{2\pi} R f(\theta) d\theta = K_m B_{n1m} F_{n1m} \cdot \sin(\beta_r - \beta_s)}$$

$K_m = L_a \frac{\pi D}{2}$ (geometrical parameters)

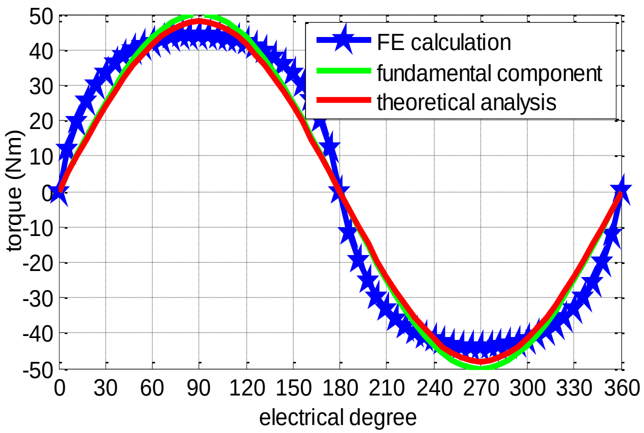
$B_{n1m} = k_B \mu_0 \frac{M h_{in}}{g + h_{in} + h_{out}}$ (peak value of fundamental magnetic field)

$F_{n1m} = k_B \; P M h_{out}$ (peak value of fundamental MMF for P pole pairs)

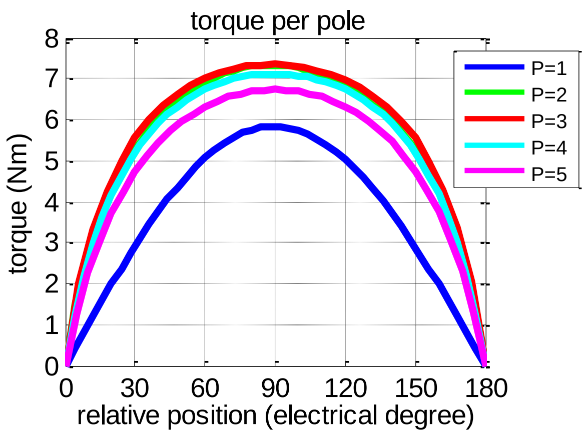
$f(\delta) = \sin(\delta) = \sin(\beta_r - \beta_s)$ (relative position between rotor and stator magnetic field)

Torque performance of MC

1. Torque comparison



2. Effect of pole pair number on torque



P	T_{max}
1	11.5
2	29
3	44
4	57
5	67

3. Observations:

- A good agreement between the results from theoretical analysis and FE calculation.
- Neglecting flux leakage between poles, the maximum torque value of MC is in a linear proportion to the pole number, which may be explained by torque expression of $T = K_m B_{n1m} F_{n1m} f(\delta)$.

Torque analysis of PM machine

1. Equivalent model:
   - Exactly the same with that of MC, thus the obtained torque expression may be applied for PM machine.

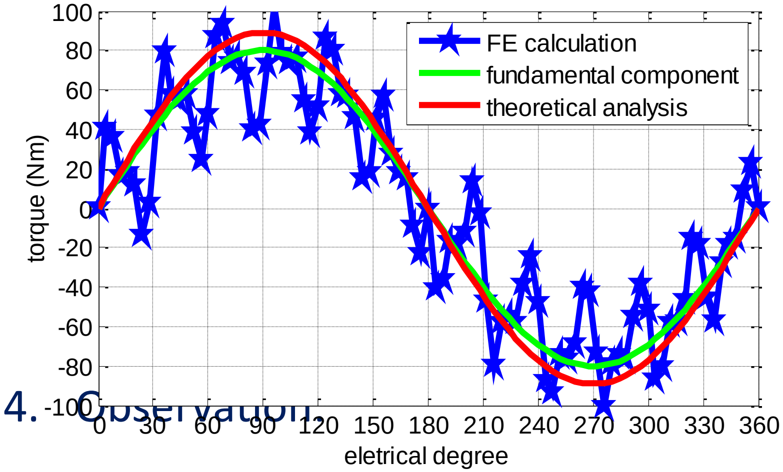
2. Specific expression of each term:
   - Stator MMF: $F_{s1\_pole}(\theta) = \frac{m}{2}\frac{4}{\pi}\frac{1}{2}\frac{JZSf_{fill}k_w}{m(2P)}\sin(P\theta - \omega t + \beta_s)$

     Thus total MMF: $F_{s1m} = PF_{s1m\_pole}$
   - Rotor magnetic field: $B_{r1}(\theta) = \mu_0 k_B \frac{Mh}{g+h}\sin(P\theta - \omega t + \beta_r)$
   - Torque expression:

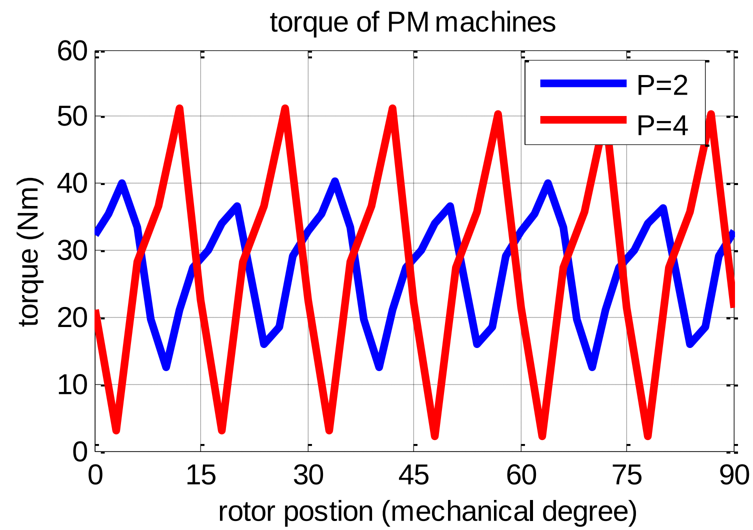
$$T = K_m B_{r1m} F_{s1m} f(\delta) = (L_a \frac{\pi D}{2})\left(\mu_0 k_B \frac{Mh}{g+h}\right)\left(\frac{JZSf_{fill}k_w}{2\pi}\right)\sin(\beta_r - \beta_s)$$

Torque analysis of PM machine

1. Torque expression validation



2. Effect of pole pair number on torque

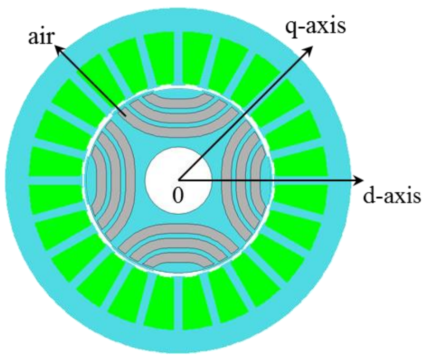


4. Observation:

- Maximum torque value of PM is determined by the main dimension (L_a and D), the rotor magnetic field production capability of per pole ($Mh$and g), and stator total current . When neglecting flux leakage between poles, the pole pair number has no effect on the total torque.

Torque analysis of SynRM

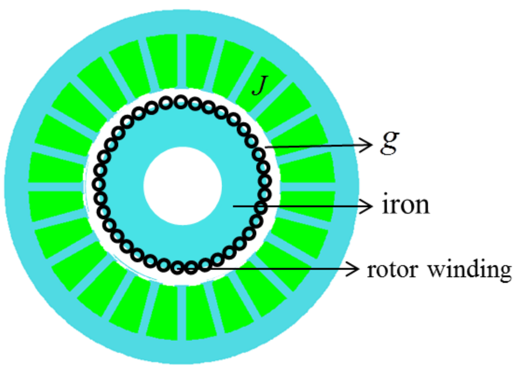
1. Model



Features:

- ➢ slot-opening stator with winding current;
- ➢ salient rotor without magnetic field source;
- ➢ non-uniform air gap;

2. Equivalent model



Features:

- ➢ identical stator configuration with that of original model;
- ➢ uniform air gap (g);
- ➢ rotor is assigned with current loading.

3. Conditions:

- ➢ Keep the stator configuration unchanged, including structure and armature current.
- ➢ Keep the air gap magnetic field unchanged.

Torque analysis of SynRM

The derived torque equation

$$T = L_a \frac{\pi D}{2} \boxed{\left(\mu_0 \frac{F_{s1m_{pole}}}{2 g_q} \left(1 - \frac{g_q}{g_d}\right) \right)} F_{s1m} \sin(2\beta)$$

2 x angle between rotor and stator magnetic field

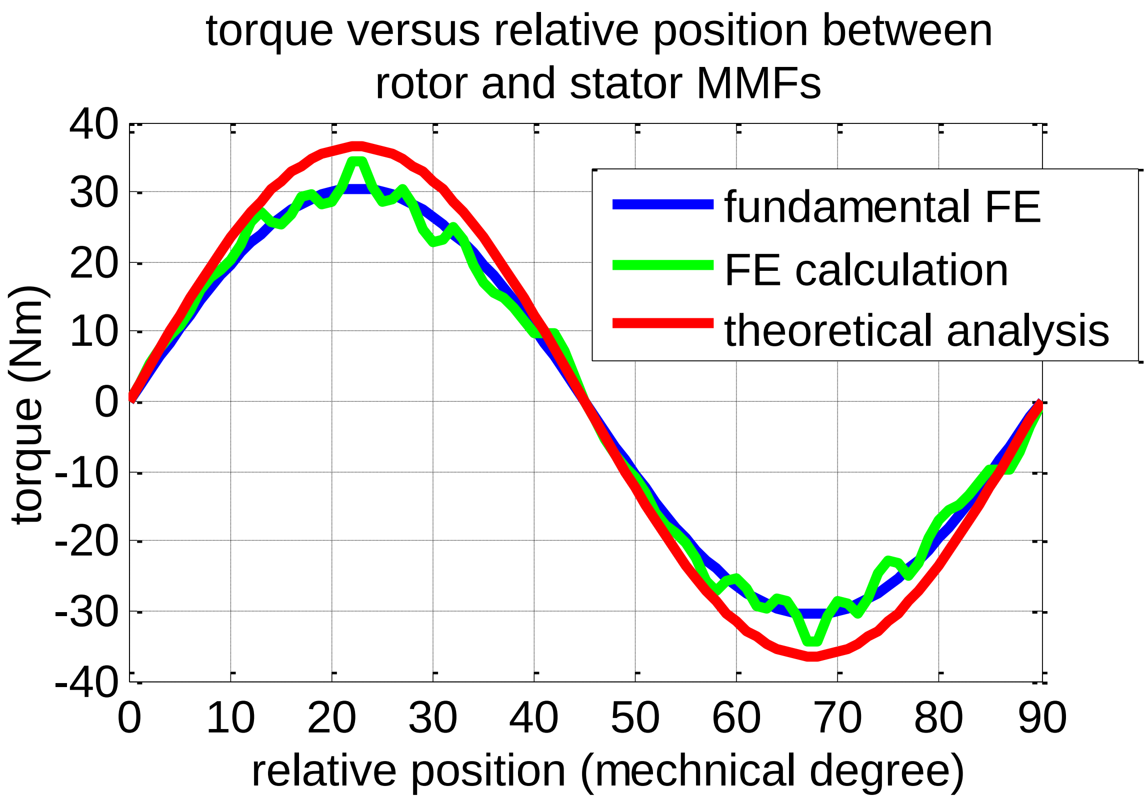
Stator total MMF (similar to PMSM)

Equivalent rotor magnetic field

$$B_{rn1m} = \mu_0 \frac{F_{s1m\_pole}}{2 g_q} \left(1 - \frac{g_q}{g_d}\right)$$

Torque analysis of SynRM

Torque comparison



Torque analysis of IM

Expressions of the terms in torque expression

- Rotor magnetic field $$B_{r1m} = \mu_0 \frac{F_{r1m}}{g}$$
- Stator MMF $$F_{s1m} = PF_{s1m\_pole}$$
- Dimensional factor $$K_m = L_a \frac{\pi D}{2}$$
- Position function $$f(\delta) = \sin(\beta_r - \beta_s)$$
- Torque calculation $$T = K_m B_{r1m} F_{s1m} f(\delta) = L_a \frac{\pi D}{2} (\mu_0 \frac{F_{s1m\_pole}}{2g}) F_{s1m} \sin 2\delta$$

Equivalent rotor magnetic field: $B_{r1m} = \mu_0 \dfrac{F_{s1m\_pole}}{2g}$

Conclusion

1. A unified torque expression is obtained for different AC machines, applying the same principle.
2. By FEM, the accuracy of this torque equation is validated;
3. Under the condition of the same stator configurations, only the peak value of rotor magnetic fields need to be compared for the torque comparison of AC machines .
   - PM machine: $B_{r1m} = \mu_0 k_B \frac{Mh}{g+h}$
   - IM: $B_{r1m} = \mu_0 \frac{F_{s1m\_pole}}{2g}$
   - SynRM: $B_{rn1m} = \mu_0 \frac{F_{s1m\_pole}}{2g_q}\left(1 - \frac{g_q}{g_d}\right)$